PG–AS-1190	**MMSS –11/ MMSS – 11C**

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

First Semester

ABSTRACT ALGEBRA

Time : 3 hours | Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words.

All questions carry equal marks.

1. If $O(G) = p^2$ where p is a prime number, then prove that G is abelian.

2. Prove that every finite abelian group is the direct product of cyclic groups.

3. State and prove the Eisenstein Criterion.

4. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

5. Let $G = S_n$ where $n \geq 5$; then show that $G^{(k)}$ for $k = 1,2,\ldots$, contains every 3-cycle of S_n.

6. If F is a field of characteristic $p \neq 0$, then show that the polynomial $x^{p^n} - x \in F[x]$, for it $n \geq 1$, has distinct roots.

7. Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

8. If $f(x)$ and $g(x)$ are two nonzero elements of $F[x]$, then show that deg $(f(x)\, g(x)) = \deg f(x) + \deg g(x)$.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words

All questions carry equal marks.

9. State and prove the third part of slows theorem.

10. State and prove the division algorithm.

11. If L is a finite extension of K and if K is a finite extension of F, then finite extension of F and show that L is a $[L:F]=[L:K]\ [K:F]$.

12. State and prove the fundamental theorem of Galois theory.

13. State and prove the Wedderburn's Theorem

PG-AS–1191 **MMSS-12/ MMSS-12C**

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

First Semester

ADVANCED CALCULUS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Define :

   (a) Uniform Continuity

   (b) Interior point

   (c) Exterior point.

2. Find the value of the Jacobian $\dfrac{\partial(u, v)}{\partial(r, \theta)}$, where $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$.

3. Explain principles of least squares.

4. If Γ is a regular curve and $f(x, y) \in C$ on Γ, then prove that $\int_{\Gamma} f(x, y)\, dx$ and $\int_{\Gamma} f(x, y)\, dy$ exists.

5. Evaluate $\iint (y - x)\, dx\, dy$ over the region R_{xy} in the xy-plane bounded by the straight lines $y = x - 3,\ y = x + 1,\ 3y + x = 5,\ 3y + x = 7$.

6. Show by examples that continuity at a point need not imply differentiability at that point.

7. Find the rectangle of parameter l which has maximum area.

8. Verify Green's theorem for $\int_{\Gamma} x^2 dx + xy dy$ where Γ is the curve given by $x = 0,\ y = 0,\ x = a,\ y = a,\ a > 0$.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

9. State and prove Euler's theorem and its converse.

10. State and prove existence theorem for implicit functions.

11. Expand the function

$f(x, y) = x^3 + 3x^2y + 4xy^2 + y^3$ by Taylor's theorem in powers of $(x-1)$ and $(y-1)$ and check by algebra.

12. State and prove Gauss's theorem.

13. Find the area of the sphere $x = a\sin\phi\cos\theta$, $y = a\sin\phi\sin\theta$, $x = a\cos\phi$, where $0 \le \theta \le 2\pi, 0 \le \phi \le \pi$.

PG-AS-1192 **MMSS-13/ MMSS-13C**

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

First Year

REAL ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words

All questions carry equal marks

1. Prove that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

2. Prove that the sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\in > 0$ there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies $|f_n(x) - f_m(x)| \leq \in$.

3. For any sequence of sets, prove that $m^k\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^k(E_i)$.

4. Show that if f is a non-negative measurable function, then $f = 0$ a.e. if, and only if $\int f\,dx = 0$.

5. Prove that a countable union of positive sets with respect to a signed positive measure v is a positive set.

6. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then prove that

   (a) $fg \in R(\alpha)$

   (b) $|f| \in R(\alpha)$ and $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$

7. If K is a compact metric space and for each $n = 1, 2, 3....$, f_n is continuous on K and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

8. Prove that every interval is measurable.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words

All questions carry equal marks

9. Prove that any continuous function on a compact metric space X is uniformly continuous on X.

10. State and prove Stone-Weierstrass theorem.

11. Prove that if $m^*(E) < \infty$ then E is measurable if, and only if, $\forall \in > 0$ there exists disjoint finite intervals $I_1, I_2, ..., I_n$ such that $m^*\left(E \,\Delta \bigcup_{i=1}^{n}\right) < \in$.

12. State and prove Lebesgue's monotone convergence theorem.

13. State and prove Riesz representation theorem for L^1.

PG-AS-1193 **MMSSE-1C/ MMSSE-1**

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

First Semester

DIFFERENTIAL GEOMETRY

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words

All questions carry equal marks

1. Find the length of the circular Helix $\vec{r}(u) = a\cos u\,\vec{i} + a\sin u\,\vec{j} + bu\,\vec{k}$ from (0, 0, 0) to (0.0.2πc). Also obtain the equation in terms of parameter s.

2. Find the equation of oscillating plane.

3. If $\vec{r} = [g(u)\cos v,\ g(u)\sin v,\ f(u)]$, find $\overrightarrow{N}$.

4. Show that the curves bisecting the angles between the parametric curves are given by $Edu^2\text{-}Gdv^2 = 0$.

5. Prove that every Helix on a cyclinder is a geodesic.

6. Find the principal direction and Principal curvature at a point on the surface $x = a(u+v)$, $y = b\,(u-v)$, $z = uv$.

7. The only compact surface of class ≥ 2 for which every point is an umblic are sphere.

8. On a paraboloid $x^2 - y^2 = z$, Find the orthogonal trajectories of the section by the planes $Z = C$.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words

All questions carry equal marks

9. State and prove Serret - Frenet formula.

10. Show that necessary and sufficient condition for a curve to be plane curve is $[r' \; r'' \; r'''] = 0$.

11. State and prove the necessary and sufficient condition for a curve $u = u(t)$, $v = v(t)$ on a surface $\vec{r} = \vec{r}\,(u,v)$ to be a Geodisic.

12. State and prove Guass - Bonnet theorem.

13. State and prove Hilbert's Theorem.

PG-AS–1194	**MMSSE-2/ MMSSE-2C**

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

First Semester

PROGRAMMING IN C++

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words.

All questions carry equal marks.

1. Elucidate the steps to create the source file in C++.

2. Distinguish between the call by reference and return by reference.

3. Describe the declaration of inline function in C++.

4. Elaborate the use of memory allocation for objects.

5. Show the declaration of arrays of objects in C++ program.

6. Highlight the rules for overloading operators.

7. Explain the constructing two-dimensional arrays with example.

8. Point out the purpose of pointers to derived classes.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words.

All questions carry equal marks.

9. Summarize the structure of C++ program.

10. Discuss the concept of function overloading with example.

11. Illustrate the constructors with default arguments in C++.

12. Compare and construct the copy constructor and dynamic constructors.

13. Outline the declaration of hierarchical inheritance in C++ program.

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PG-AS-1195	**MMSS-21**

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Second Semester

APPLIED MECHANICS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Derive the components of velocity and acceleration in cylindrical coordinates.

2. Verify the principle of conservation of energy for a particle falling freely under gravity.

3. Find the kinetic energy of rotation of a rigid body with respect to the principal axes terms of Eulerian angles and interpret the result when $A = B$.

4. A particle of mass m is attracted to a fixed point O by an inverse square force $-\frac{\mu m}{r^2}$, where μ is the gravitational constant. Using the polar coordinates (r, θ), find the Hamilton' s canonical equations of motion.

5. Find the generating function for the transformation $p = \frac{1}{q'} D \; q = p' q'^2$.

6. Find the kinetic energy of a rigid body with respect to a fixed point.

7. A particle of mass m can slide without friction on the inside of a small tube which is bent in the form of the circle of radius r. The tube rotates about a vertical diameter with a constant angular velocity ω. Find the differential equation of motion.

8. State and prove Liouville's theorem for one degree of freedom.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

9. (a) Discuss the angular momentum of a system of particles.

   (b) Find the moment of inertial of a spherical shell about its diameter.

10. A rectangular plate spins with constant angular velocity ω about a diagonal. Find the couple which must out on the plate in order to maintain this motion.

11. Discuss the steady precession of a top.

12. Derive Lagrange's equation for a particle in a plane.

13. Discuss Hamilton's principle.

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PG–AS–1196 **MMSS–22**

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Second Semester

COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. If $f(z)$ is analytic in an open disk Δ, then prove that $\int_{\gamma} f(z)\,dz = 0$ for every cycle γ in Δ.

2. Prove that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycle γ in Ω and all points a which do not belong to Ω.

3. If $g(z)$ is an entire function, then prove that $f(z) = e^{g(z)}$ is entire and not equal to zero.

4. State and Prove Harnack's inequalities for harmonic functions.

5. Prove that any two bases of period module are connected by a unimodular transformation.

6. State and Prove fundamental theorem of algebra.

7. State and Prove Argument principle.

8. Prove that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}(1 - p_n^{-s}) \, f\,or\,\sigma = \mathrm{Re}(s) > 1$.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

9. (a) Suppose that $\varphi(\zeta)$ is continuous on the arc γ. Then prove that the function $F_n(z) = \int_{\gamma} \frac{\varphi(\zeta)}{(\zeta - z)^n} d\zeta$ is analytic in each of regions determined by γ, and its derivative is $F_n'(z) = n F_{n+1}(z)$.

   (b) Evaluate $\int_{|z|=2} \frac{dz}{z^2 + 1}$.

10. Derive Poisson's formula for Harmonic functions.

11. State and Prove Mittag-Leffler's theorem for Meormorphic function.

12. State and Prove Riemann Mapping theorem.

13. (a) Prove that

$$\wp(z+u) = -\wp(z) - \wp(u) + \frac{1}{4}\left[\frac{\wp'(Z)-\wp'(u)}{\wp(Z)-\wp(u)}\right]^2.$$

(b) Prove that an elliptic function without poles is a constant.

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PG-AS-1197	MMSS-23

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Second Semester

LINEAR ALGEBRA

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Show that every n-dimensional vector space over the field F is isomorphic to the space F^n.

2. Prove that a polynomial f of degree n over a field F has at most n roots in F.

3. Let K be a commutative ring with identity, and let A and B be an $n \times n$ matrices over K. Then prove that det (AB) = (det$A$) (det$B$).

4. Let W be an invariant subspace for T. The characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T. Then prove that the minimal polynomial for T_w divides the minimal polynomial for T.

5. Let F be a field and Let B be an $n \times n$ matrix over F. Then Prove that B is similar over the field F to one and only one matrix which is in rational form.

6. Find the dual basis of the basis $B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for V.

7. Using Cramer's rule, solve the following system of linear equations

$$x + y + z = 11$$

$$2x - 6y - z = 0$$

$$3x + 4y + 2z = 0$$

8. Define:

   (a) Cyclic vector.

   (b) Companion matrix.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

9. Let V and W be vector space over the field F and let T be a linear transformation from V into W. Suppose that V is finite dimensional, then prove that rank (T) + nullity (T) = dim(V).

10. Let F be a characteristic zero and f a polynomial over F with deg $f \leq n$. Then prove that the scalar c is a root of multiplicity r if and only if

$$\left(D^k f\right)(c) = 0,\ 0 \leq k \leq r-1$$

$$\left(D^r f\right)(c) \neq 0$$

11. Diagonalize the matrix $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

12. State and Prove the Primary decomposition theorem.

13. State and Prove Cyclic Decomposition theorem.

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PG-AS-1198 **MMSSE-3**

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Second Semester

PARTIAL DIFFERENTIAL EQUATION

Time : 3 hours Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions out of Eight questions in 300 words

All questions carry equal marks

1. Solve $x(y-z)p + y(z-x)q = z(x-y)$.

2. Find the integral surface of the PDE $(x-y)p + (y-x-z)q = z$ through the circle $z = 1, x^2 + y^2 = 1$.

3. Find the complete integral of $(p_1 + x_1)^2 + (p_2 + x_2)^2 + (p_3 + x_3)^2 = 3(x_1 + x_2 + x_3)$.

4. Solve $(D^3 - 4D^2D' + 4DD'^2) = 2\sin(3x + 2y)$.

5. Solve $(D^3 - 7D'^2 D + 6D'^3) = x^2 + xy^2 + y^3$.

6. Find the general solution of heat flow equation $k\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial u}{\partial t}$ by method of separation of variables.

7. A string of length l is fixed at both ends. The motion is started by displacing the string into the form of a curve $y = kx(l - x)$. Find the displacement function $y(x, t)$.

8. Solve the on dimensional wave equation using method of separation of variables.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions out of Five questions in 1000 words

All questions carry equal marks

9. Find the complete integral of $p^2 - y^2 q = y^2 - x^2$.

10. Solve $\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial u}{\partial t}$ $0 < x < 4, t > 0$, subject to the conditions $u(0, t) = u(4, t) = 0$, $u(x, 0) = 2x$.

11. Solve $z = px + qy + \sqrt{p^2 + q^2 + 1}$.

12. Solve $(3D^2 - 2D'^2 + D - 1) = 4e^{x+y}\cos(x+y)$.

13. Find the steady state temperature distribution of a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions $u(0, y) = u(a, y) = 0,\ 0 \le y \le b,\ u(x, b) = 0$, and $u(x, 0) = x(a - x)\ 0 \le x \le a$.

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PG–AS–1199	MMSSE–4

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Second Semester

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words.

All questions carry equal marks.

1. A random sample (x_1, x_2, x_3, x_4) of size 4 is drawn from a normal population unknown mean μ. Consider the following estimators to estimate

   (a) $T_1 = \dfrac{x_1 + x_2 + x_3 + x_4}{4}$

   (b) $T_2 = \dfrac{x_1 + x_2 + x_3}{4} + x_4$

   Are T_1 and T_2 are unbiased?

2. If t_1 is a most efficient estimator and t_2 is an unbiased estimator with efficiency e and if the correlation coefficient ρ between t_1 and t_2 is ρ, show that $\rho = \sqrt{e}$.

3. Time taken by workers in performing a job are given below :

Type I	21	17	27	28	24	23	–
Type II	28	34	43	36	33	35	39

Test whether there is any significant difference between the variances of time distribution.

4. The mean life time of a sample of 100 light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours, against the alternative hypothesis $\mu \neq 1600$ hours with α = 0.05 and 0.01?

5. Find the moment generating function of $\aleph^2$ - distribution (chi-Square).

6. The following is the Latin square layout of a design, when four varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion.

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

7. Find the covariance matrix for the two random variable $X1$ and $X2$ whose joint probability function is given below.

$X2$ \ $X1$	0	1	$P(X1)$
–1	0.24	0.6	0.3
0	0.16	0.14	0.3
1	0.4	0	0.4
$P(X2)$	0.8	0.2	1

8. Suppose $\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & -25 \end{bmatrix}$. Find $V^{\frac{1}{2}}$ and ρ.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words.

All questions carry equal marks.

9. Two horses A and B were tested according to the time (in sec) to run a particular race with following Results.

   Horse A : 28 30 32 33 33 29 34

   Find the moment generating function of

   Test whether the horse A is running faster than B at 5% level of significance and also find 99% confidence interval for the difference in mean.

10. Using the data given in the following table to test at the 0.01 level of significance whether a person's ability in mathematics is independent of his/her interest in Statistics.

		Ability in Mathematics		
		Low	Average	High
Interest in Statistics	Low	63	42	15
	Average	58	61	31
	High	14	47	29

11. The following data represent a certain person to work from Monday to Friday by four different routes.

Routes	Days				
	Mon	Tue	Wed	Thu	Fri
1	22	26	25	25	31
2	25	27	28	26	29
3	26	29	33	30	33
4	26	28	27	30	30

Test at the 0.05 level of significance whether the differences among the means obtained for the different routes are significant and also whether the differences among the means obtained for the different days of the week are significant.

12. State and prove Cramer – Rao inequality.

13. Suppose that X_1, X_2 and X_3 have covariance matrix $\sum = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Calculate the population principal components.

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PG–AS–1200	**MMSS–31**

P.G. DEGREE EXAMINATION – JULY 2024.

Mathematics

Third Semester

TOPOLOGY

Time : 3 hours | Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let X be a set; let $\mathfrak{B}$ be a basis for a topology τ on X. Then prove that τ equals the collection of all unions of elements of $\mathfrak{B}$.

2. State and prove Uniform limit theorem.

3. Prove that the image of a connected space under a continuous map is connected space.

4. Prove that every closed subspace of a compact space is compact.

5. Prove that a subspace of completely regular space is completely regular.

6. Let Y be a subspace of X; let A be a subset of Y; let $\overline{A}$ denote the closure of A in X. Then prove that the closure of A in Y equals $\overline{A} \cap Y$.

7. Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

8. State and Prove sequence lemma.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

9. (a) Let $\mathfrak{B}$ and $\mathfrak{B}'$ be bases for the topologies τ and τ' respectively on X. Then prove that the following are equivalent :

   (i) τ' is finer than τ.

   (ii) For each $x \in X$ and each basis element $B \in B$ containing x, there is a basis $B' \in B'$ such that $x \in B' \subset B$.

   (b) Define :

   (i) Order topology

   (ii) The subspace topology.

10. (a) Let X be a topological space. Show that the following conditions hold :

(i) ϕ and X are closed.

(ii) Arbitrary intersections of closed sets are closed.

(iii) Finite unions of closed sets are closed.

(b) State and Prove the Pasting lemma.

11. If L is a linear continuum in the order topology then prove that L is connected and so are intervals and rays in L.

12. Prove that the product of finitely many compact spaces is compact.

13. State and Prove Urysohn lemma.

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PG-AS-1201	MMSS-32

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Third Semester

FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions

1. If N is a normed linear space with respect to the distance defined as $d(x, y) = \|x - y\|$, then prove that N becomes a metric space.

2. State and Prove closed graph theorem.

3. State and Prove Polarization identify.

4. Prove that if T_1 and T_2 be self adjont operators then their product T_1T_2 is self adjoint if and only if $T_1T_2 = T_2T_1$.

5. If T is a normal operator on a Hilbert space H, then prove that the eigen space of T are pairwise orthogonal.

6. State and Prove Cauchy's inequality.

7. Prove that a non-empty subset X of a normed linear space N is bounded if and only if $f(X)$ is a bounded set of numbers for each f in N^*.

8. If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then the linear subspace $M + N$ is also closed.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions

9. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset $x + M$ in the quotient space N / M is defined by $\|x+m\| = \inf \{\|x+m\|; m \in M\}$, then prove that N / M is a normed linear space. Further, if N is a Banach space then so is N / M.

10. Le M be a linear subspace of a normed linear space N, and let f be a functional defined on M. If x_0 is a vector not in M, and if $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0 then prove that f can be extended to function f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

11. State and Prove Riesz representation theorem.

12. Let T be an operator on a Hilbert space H, then prove that there exists a unique operator T^* on H such that for all $x, y, \in (Tx, y) = (x, T^* y)$.

13. Prove that two matrices in A_n are similar if and only if they are matrices of a single operator on H relative to (possibly) different bases.

PG-AS-1202	MMSS-33

P.G. DEGREE EXAMINATION – JULY 2024.

Mathematics

Third Semester

ORDINARY DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Solve the IVP $y'' - 2y' - 3y = 0,\ y(0) = 0,\ y'(0) = 1$.

2. Show that the n solution of the homogenous equation of order n is given by $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + ... + a_n y = 0$ are linearly independent on any interval I.

3. Use annihilator method to find a particular solution of $y'' + 4y = \cos x$.

4. Find the singular points of the equation $(1-x)^2 y'' - 2xy' + 2y = 0$ and determine whether they are regular singular points or not.

5. Solve the equation $y' = \dfrac{3x^2 - 2xy}{x^2 - 2y}$ after checking the exactness.

6. Find the solution of the initial value problem $y'' + (3i - 1)y' - 3y = 0,\ y(0) = 0;\ y'(0) = 1$.

7. If $P_n(t)$ is a Legendre Polynomials, then Prove that $\int_{-1}^{1} P_n^2(t)\,dt = \dfrac{2}{2n+1}$.

8. Show that $f(x, y) = x\,y^2$ satisfies Lipschitz condition on the rectangle. $|x| \le 1$, $|y| \le 1$ but does not satisfy a Lipschitz condition on the strip $|x| \le 1,\ y < \infty$.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

9. State and prove the existence and uniqueness of the solution of the initial value problem involving second order homogenous ordinary differential equation with constant coefficients.

10. Consider the equation $y''' - 4y' = 0$

 (a) Compute three linearly independent solutions.

 (b) Compute the wronskian of the three linearly independent solutions.

 (c) Find that ϕ satisfying $\phi(0) = 0, \phi'(0) = 1, \phi''(0) = 0$.

11. Derive Rodrigue' s formula.

12. Obtain two linearly independent solution of $x\,y'' + y' - y = 0$ which are valid near $x = 0$.

13. State and Prove the necessary and sufficient condition for an equation $M(x,y)\,dx + N(x,y)\,dy = 0$ to be exact.

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PG–AS–1203	**MMSS–34**

P.G. DEGREE EXAMINATION — JULY 2024

Mathematics

Third Semester

NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve the following system of equations by Gauss Elimination method

$$y + 2z = 5$$
$$x + 2y + 4z = 11$$
$$-3x + y - 5z = -12$$

2. Obtain a quadratic polynomial approximation to $f(x) = e^{-x}$ using Lagrange's interpolation method, taking three points $x = 0, \frac{1}{2}, 1$.

3. A particle is moving along a straight line. The displacement x at some time instances t are given below:

t :	0	1	2	3	4
x :	5	8	12	17	26

Find the velocity and acceleration of the particle at $t = 4$.

4. By using Modified Eulers method to determine the value of y when $x = 0.1$ and 0.2 given that $y(0) = 1$ and $y' = x^2 - y$.

5. Investigate the stability of the parabolic equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ which is approximated by finite difference scheme.

6. Discuss the Convergence of Gauss-Seidal's method.

7. Find the value of y when $x = 1.5$ from the following table using Newton's divided difference formula.

x :	1	5	7	10	12
y :	0.6931	1.7918	2.0794	2.3979	2.5649

8. Evaluate $\int_0^2 (2x - x^2)dx$, taking 6 intervals, by

(a) Trapezoidal rule and

(b) Simpson's 1/3 rule.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions.

9. Solve the following system of linear equations by Gauss-Jacobi's method correct up to four decimal places and calculate the upper bound of absolute errors.

$27x + 6y - z = 54$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

10. Derive Lagrange's interpolating polynomial.

11. Find the least squares approximation polynomial of degree two for the function $f(x) = \sin \pi x$ on the interval $[0, 1]$.

12. Solve the following IVP $y'' - y = x$ with $y(0) = 0$ and $y''(0) = 1$ using finite difference method for $x = 0.01, 0.02, \ldots, 0.10$.

13. Use the Crank-Nicolson method to calculate a numerical solution of the problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0$ Where $u(0, t) = u(1, t) = 0$, $u(x, 0) = 2x, t = 0$. Mention the value of $u\left(\frac{1}{2}, \frac{1}{8}\right)$ by taking $h = \frac{1}{2}, \frac{1}{4}$ and $k = \frac{1}{8}, \frac{1}{16}$.

PG–AS–1204 **MMSSE–5**

P.G. DEGREE EXAMINATION — JULY 2024

Mathematics

Third Semester

GRAPH THEORY

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Let $G = (V, E)$ be a non-empty, non-trivial graph. Then prove that G has atleast one pair of vertices with equal degree.

2. Prove that for any graph G, $\kappa \le \kappa' \le \delta$.

3. Explain Konigsberg Brige problem.

4. Prove that in a critical graph, no vertex cut is a clique.

5. Prove that $K_{3,3}$ is nonplanar.

6. Prove that in a graph G with vertices u and v, every u-v walk contains a u-v path.

7. Prove that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by atleast two internally disjoint paths.

8. Prove that a connected graph has an Euler trial if and only if has at most two vertices of odd degree.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions.

9. (a) Prove that a graph G is bipartite if and only if it contains no cycles of odd length.

   (b) Prove that a connected graph is a tree if and only if every edge is a cut edge.

10. Let G be a connected graph with atleast three points. Then prove that the following statements are equivalent:

    (a) G is a block

    (b) Any two points of G lie on a common cycle.

    (c) Any point and any line of G lie on a common cycle.

11. (a) State and Prove Hall's theorem.

    (b) Write Fleury's algorithm.

12. State and Prove Vizing's theorem.

13. (a) Prove that a graph has a dual if and only if it is planar.

    (b) State and Prove Euler's formula.

PG–AS-1205	MMSS–41

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

Fourth Semester

INTEGRAL TRANSFORMS AND CALCULUS OF VARIATIONS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words.

All questions carry equal marks.

1. Find the Laplace transforms of the following functions

   (a) $L\left(te^{-t}\sin 2t\right)$

   (b) $L\left(e^{-2t}\int_0^t \sin 2t\,dt\right)$.

2. State and prove initial and final value theorem in Laplace transforms.

3. Find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$.

4. State and prove convolution theorem in Fourier transforms.

5. Find the Fourier sine transforms of $\frac{e^{-ax}}{x}$ and hence find Fourier Sine transforms of $\frac{1}{x}$.

6. Obtain the Euler's equation for the extremal of the functional $\int_{x1}^{x_2}\left(y^2 - yy' + (y')^2\right)dx$.

7. Show that the sphere is the solid figure of revolution which for a surface area has maximum volume.

8. Evaluate the integral using Laplace transforms $\int_0^\infty t^3 e^{-t} \sin t \, dt$.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1,000 words.

All questions carry equal marks.

9. Derive the Euler's equation in calculus of variation.

10. Find the Fourier Transform of $f(x) = \begin{cases} a-|x| & if \quad |x| \le a \\ 0 & if \quad |x| > a \end{cases}$. Hence prove that $\int_0^\infty (\sin t/t)^4 \, dt = \pi/3$.

11. Solve by using Laplace transforms $y'' - 4y' + 3y = e^{-t}$ given $y(0) = 1$, $y'(0) = 0$.

12. Find the Laplace transforms of the periodic function with period $2b$ and which is defined as follows

$$f(t) = \begin{cases} t & 0 < t < b \\ 2b - t & 0 < t < 2b \end{cases}$$

13. Find the extremal of the function $\int_0^1 (y'^2 - x^2 + \lambda y)dx$. Under the condition $y(0) = y(1) = 0$ and subjected in the constraint $\int_0^1 y dx = \frac{1}{6}$.

PG–AS–1206	**MMSS–42**

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

Fourth Semester

PROBABILITY AND RANDOM PROCESSES

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words.

All questions carry equal marks.

1. The daily consumption of milk in excess of 20000 litres in a town is approximately exponentially distribution with parameter 1/3000. The town has a daily stock of 35,000 litres. What is the probability that of 2 days selected at random the stock is insufficient for both days?

2. In company producing lenses there is a small chance of 1/500 for any lenses to be defective, the lenses are supplied in pocket of 10 each. Use Poisson distribution find the appropriate number of packets containing (a) no defective, (b) at least 2 defective (c) atmost 3 defective (d) more than 2 defective in a consignment of 10,000 pockets.

3. The probability mass function is given by $P(x, y) = k(2x + 3y)$, $x = 0,1,2;\ y = 1,2,3$. Find all the marginal and conditional probability function

4. The joint pdf of a two dimentional random variable (X, Y) is given by

$f(x, y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$ compute $P\left(X > 1 / Y < \frac{1}{2}\right), P\left(Y < \frac{1}{2}\right)$.

5. Prove the Sum of two independent Poisson process is again a Poisson process.

6. If the random process $\{X(t)\}$ is given by $X(t) = 10 \cos(100\ t + \theta)$, here θ is uniformly distributed over (0.2π), prove that $\{X(t)\}$ is a WSS process.

7. Obtain the equation of the lines of regression from the following data:

X	1	2	3	4	5	6	7
T	9	8	10	12	11	13	14

8. State and prove Chebyshev's inequality.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1,000 words.

All questions carry equal marks.

9. Derive moment generating function of Binomial distribution and hence find its mean and variance.

10. If X and Y are independent RVs with pdf's $e^{-x}; x \geq 0$ and $e^{-y}; y \geq 0$, respectively, find the pdf's of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent?

11. A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if '6' appeared. Find the probability that he takes a train on the third day. Also find the probability that he drives to work in the long run.

12. The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3...$

having three states 1, 2 and 3 is $p = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7 \ 0.2 \ 0.1)$.

Find $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$ and $P\{X_2 = 3\}$.

13. Prove that inter arrival time of a Poisson process follows exponential distribution with mean $1/\lambda$. Suppose that customers arrive at a counter according to a Poisson process with mean rate of 2 per minute. Find the probability that the interval between two successive arrivals is (a) more than one min (b) between 1 and 2 mm (c) 4 min or less..

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PG–AS-1207	**MMSS–43**

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

Fourth Semester

CONTINUM MECHANICS

Time : 3 hours   Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Define:

   (a) Transpose of a tensor

   (b) Symmetric and Antisymmetric tensor.

2. Given the velocity field $v_i = \frac{-x_i}{1-t}$. Find the density of a material particle as a function of time.

3. Obtain the components of stress tensor.

4. For an isotropic material,
   (a) Show that the principal direction of stress and strain coincide.
   (b) Find a relation between the principal values of stress and strain.

5. Explain Laminar and turbulent flow.

6. Discuss Principal strain

7. Discuss compatibility condition for rate of deformation.

8. Find the vorticity vector for the simple shearing flow $v_i = kx_2, v_2 = v_3 = 0$.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

9. Obtain principal values and principal directions of real symmetric tensors.

10. Given the velocity field $v_1 = kx_2 \;\; v_2 = v_3 = 0$.
   (a) Find the rate of deformation and spin tensor.
   (b) Determine the rate of extension of the material elements $dx^{(1)} = (ds_1)\, e_1$,
   $$dx^{(2)} = (ds_2)\;\; e_2 \text{ and } dx = \frac{ds}{\sqrt{5}}(e_1 + 2e_2)$$
   (c) Find the maximum and minimum rates of extension.

11. The stress field for the Kelvin's problem (an infinite elastic space loaded by a concentrated load at the origin) is given by the following stress components in cylindrical coordinates:

$$T_{rr} = \frac{Az}{r^3} - \frac{3r^2 z}{R^5},\ T_{\theta\theta} = \frac{Az}{r^3},\ T_{zz} = -\left[\frac{Az}{r^3} + \frac{3r^2 z}{R^5}\right].$$

$$T_{rz} = T_{rr} = \frac{Az}{r^3} - \frac{3r^2 z}{R^5}, T_{\theta\theta} = \frac{Az}{r^3}, T_{zz} = -\left[\frac{Az}{R^3} + \frac{3rz^2}{R^5}\right],$$

$$T_{z\theta} = T_{r\theta} = 0$$

Where $R^2 = r^2 + z^2$.

and A is constant. Verify that the given state of stress is in equilibrium in the absence of body forces.

12. Discuss equations of the infinitesimal theory of elasticity

13. Obtain the solution for Hagen-Poiseuille flow.

PG–AS–1208	MMSS–44

P.G. DEGREE EXAMINATION – JULY, 2024.

Mathematics

Fourth Semester

MATHEMATICAL METHODS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Solve the homogeneous Fredholm integral equation $g(s) = \lambda \int_0^1 e^s e^t g(t)\, dt$.

2. Solve the integral equation $f(s) = \int_s^b \frac{g(t)\, dt}{(t^2 - s^2)^\alpha}$, $0 < \alpha < 1;\ a < s < b$.

3. Find the Fourier sine transform of $\frac{1}{t} e^{-at}$, $a > 0$.

4. Find the Hankel transform of $\frac{dt}{dx}$ when $f = \frac{e^{-ax}}{x}$ and $n = 1$.

5. State and prove the first fundamental lemma of calculus of variations.

6. Solve the integral equation by iterative method $g(s) = 1 + \lambda \int_0^{\pi} \sin(s+t)\, g(t)\, dt$.

7. State and Prove the shifting property of Fourier transforms.

8. Find the Hankel transform of $\frac{\sin ax}{x}$ taking $x J_0(px)$ as the kernel of the transformation.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

9. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s+t)\, g(t)\, dt$.

10. Obtain Poincare-Bertrans transformation formula.

11. Solve the following boundary value problem in the half-plane

$$u_{xx} + u_{yy} = 0, -\infty < x < \infty,\ y > 0$$

$$u(x,0) = f(x), -\infty < x < \infty$$

12. Obtain the relationship between Fourier transform and Hankel transforms.

13. Discuss Brachistochrone problem.

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PG–AS-1209 **MMSSE–6**

P.G. DEGREE EXAMINATION — JULY 2024.

Mathematics

Fourth Semester

OPTIMIZATION TECHNIQUES

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Find the initial basic feasible solution for the following transportation problem by VAM.

	I	II	III	IV	
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
	200	225	275	250	

2. Define :

   (a) Mixed Integer Programming Problem.

   (b) Zero-One Integer Programming Problem

3. Solve the following problem by dynamic programming

   Maximize $z = 2x_1 + 5x_2$

   $2x_1 + x_2 \leq 43$

   $2x_2 \leq 46$

   $x_{1,}, x_2 \geq 0$

4. Explain the operating characteristics of queue system.

5. Find the maxima and minima of the function

   $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56$

6. Explain Hungarian Method for solving assignment problem.

7. Write the procedure for solving 0-1 integer programming problem.

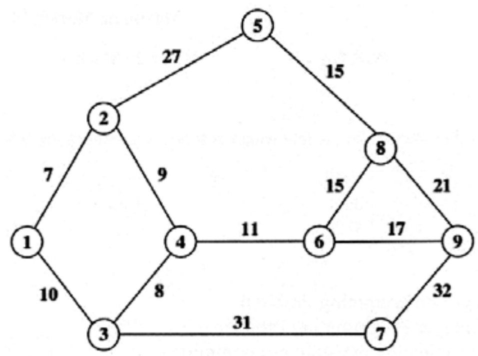
8. An oil engine manufacturer purchases lubricants at the rate of Rs.42 per piece from a order. The requirement of these lubricants is 1,800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs.16 and inventory carrying charges per rupee per year is only 20 paise.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

9. Find the shortest path from node 1 to node 9 of the distance network shown in figure below using Dijkstra's algorithm.



10. Find the optimum integer solution to the following linear programming problem:

Maximize $Z = 5x_1 + 8x_2$

Subject to :

$x_1 + 2x_2 + \leq 8$

$4x_1 + x_2 \leq 10$

$x_1, x_2 \geq 0$ and are integers.

11. Use dynamic programming to show that $p_1 \log p_1 + p_2 \log p_2 + \cdots + p_n \log p_n$

Subject to the constraint $p_1 + p_2 + \cdots + p_n$ and $p_i \geq 0$ for all i is minimum $p_1 = p_2 = \cdots = p_n = \frac{1}{n}$.

12. Discuss deterministic inventory problem with shortages.

13. Solve by Lagrangean method

$\min f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$

Subject to

$g_i(x_1, x_2, x_3) = x_1 + x_2 + 3x_3 = 2$

$g_2(x_1, x_2), x_3 = 5x_1 + 2x_2 + x_3 = 5$.

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